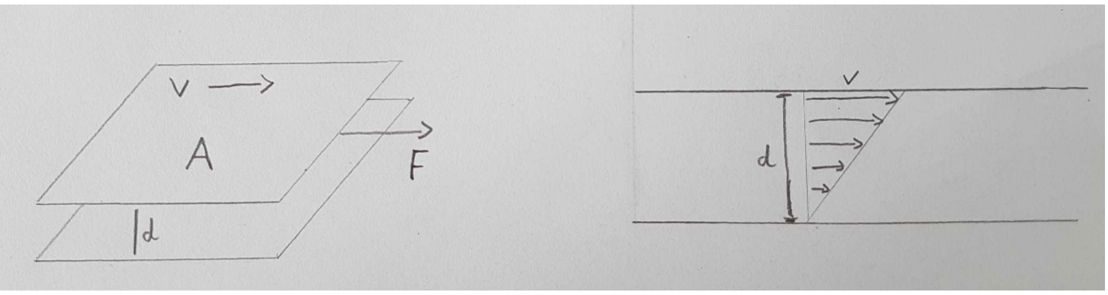
Stokes-Einstein relation

What determines diffusion constants?

- size: The bigger, the slower $\rightarrow$ radius [length]
- temperature: The hotter the faster $\rightarrow$ kT [energy]
- medium: What properties of the medium might matter?

Viscosity

Particles diffuse more slowly in more viscous medium, e.g. honey compared to water. So what exactly is viscosity? If you are curious, read up on viscosity on wikipedia.



$$F \sim \frac{Av}{d} \quad \Rightarrow \quad F = \eta \frac{Av}{d}$$

Viscosity η is the proportionality constant linking movement of molecules to frictional force.

Viscosity has dimension

$$[\eta] = \left[\frac{Fd}{Av}\right] = \frac{energy \times time}{volume} = \frac{force \times time}{area}$$

Viscosity is typically measured in Ns/m^2 and relevant values for us are

- water: $0.001\frac{Ns}{m^2}$
- cytosol: $0.003\frac{Ns}{m^2}$

Coming back to our quest of understanding diffusion constants:

- size: The bigger, the slower $\rightarrow$ radius [length]
- temperature: The hotter the faster $\rightarrow$ kT [energy]
- viscosity: [energy time/volume]

Diffusion constants have dimension [area/time]. How do you combine the above to obtain this dimension?

$$D \sim \frac{kT}{r\eta}$$

Dimensional analysis suggests that kT, r, and η should combine as above. And this is the correct answer up to a numerical prefactor. Careful calculation yields the **Stokes-Einstein relation**:

$$D = \frac{kT}{6\pi r\eta}$$

The remarkable fact about this equation is that combines **macroscopic** quantities like viscosity and temperature to make predictions about a **microscopic** quantity, namely the diffusion constant of a molecule.

Rule of thumb

We will typically deal with temperatures around 300K (room temperature, $kT \approx 4pN \times nm$) and are interested in the biological questions such that $\eta \approx 0.003Ns/m^2$:

$$D = \frac{4pN \times nm}{6\pi 0.003Ns/m^2}\frac{1}{r} \approx \frac{\mu m^3}{15s}\frac{1}{r}$$

We can use this rule of thumb to estimate diffusion coefficients of objects in cells. For a protein with radius 3nm, for example, we obtain $D \approx 20\mu m^2/s$.

Dig deeper

- Look up the viscosity of honey! How smaller would diffusion be? How long would it take a protein to diffuse $10\mu m$ in honey?
- How well do the diffusion constants we discussed in previous lectures conform with the Stokes-Einstein relation?

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