Cytokeleton

- Actin: 7nm, two twisted protofilaments
- Intermediate filaments: 10nm, coiled-coil
- Microtubule: 25nm, hollow tube

**Microtubule**

→ projector

**Actin**

→ projector

- Keratocyte mobility

![Diagram of cell motion with nucleus, lamellipodium, actin network, adhesion, and zones of polymerisation and disassembly.]

- Little myosin at the front
  → force by polymerisation
- Myosin at the rear
  → driving flux
Propulsion of *listeria*

![Diagram showing a bacterium with forces labeled as $F_{end}$, $F_a$, and $f_w$.]

\[ F_{end} + a F_a = w f_w \]

\[ \text{# of working filaments} \] \[ \text{# of attached filaments} \]

\[ \frac{da}{dt} = n - \frac{F_a}{k} \]

\[ \frac{dw}{dt} = \frac{F_a}{k} - \lambda w \]

\[ \bar{a} = \frac{n}{k} \]

\[ \bar{w} = \frac{n}{k \lambda} \]

- How do rates depend on forces?

\[ V = V_{max} e^{-\frac{fu}{kT}} - V_{dep} \]

\[ \text{reduction of polymerization due to force } f_w \]

\[ \text{gap } d \text{ is } \]

Boltzmann distributed
- dissociation is faster for large pulling forces
- as the cell moves, the force increases

\[ F = k V t \Rightarrow s(V, t) = s_0 e^{k V t / F_0} \]

\( F_0 \) is a rupture scale of the attachment bond

\[ p(t) = s(s(t)) e^{-s(t) / s(t)} \]

not ruptured yet

\[ s_0 e^{k V t / F_0} - s(t) = s_0 e^{k V t / F_0} - s(t) \]

not ruptured yet

\[ \text{for} \ V > \frac{s_0 F_0}{K} = V_0 \]
most ruptures happen when stretched

\[ V < V_0 = \text{unstretched} \]

\[ p(t) = -\frac{d}{dt} s(t) e^{k V t / F_0} [1 - e^{k V t / F_0}] \]
the average attachment force given \( v = \frac{V}{\nu} \)

\[
f_a = k v \langle \tau \rangle = k v v \int_0^1 \frac{d}{d\xi} \left[ \frac{1}{2} \left( 1 - e^{-\nu \xi} \right) \right] \]

\[
= \frac{f_b}{\nu} \int_0^1 \frac{d}{d\xi} \left( 1 - e^{-\nu \xi} \right) = \frac{f_b}{\nu} \omega(v)
\]

\[
\approx \begin{cases} \frac{f_b}{\nu} v & v \ll 1 \\ \frac{f_b}{\nu} \log v & v \gg 1 \end{cases}
\]

**Putting things back together**

\[
V = V_{\max} e^{\frac{-5wL}{\nu T}} - V_{\text{dep}}
\]

\[
F_\text{e} + \frac{n}{8} f_a = \frac{n}{k} f_w \Rightarrow f_w = \frac{k}{n} F_\text{e} + \frac{k}{n} f_a
\]

\[
= \frac{k}{n} F_\text{e} + \frac{k}{n} \frac{f_b}{\nu} \omega(v)
\]

\[
\Rightarrow V = V_{\max} e^{\frac{-5wL}{\nu T} \left[ \frac{F_\text{e}}{k} + \frac{f_b}{\nu} \omega(v) \right]} - V_{\text{dep}}
\]

\[
\varepsilon_1 = \frac{F_\text{e}}{k T} \frac{\omega}{\omega_0}
\]

\[
\varepsilon_2 = \frac{V_{\max}}{V_0}
\]

\[
\varepsilon_3 = \frac{V_{\text{dep}}}{V_0}
\]

\[
\varepsilon_4 = \frac{F_\text{e}}{k T} \frac{k}{n} \frac{\omega}{\omega_0}
\]

\[
\Rightarrow \quad V = \varepsilon_4 e^{\frac{-x \nu \omega^2(v)}{\omega_0} - \varepsilon_4}
\]
Euler buckling instability

Persistence length of microtubuli?

- \( \ell_p = \frac{L}{4T} = 1 \text{mm} \Rightarrow \text{stress} \times x = 4\rho N \text{mm } \text{mm} = 4\rho N \text{mm}^2 \)

- How much compression can a microtubule support?

\[
\Theta = \frac{L}{R^2} \\
X = L - 2R \sin \frac{\Theta}{2} = L(1 - \frac{1}{2} \Theta \sin \frac{\Theta}{2})
\]
\[ E_{\text{buend}} = \frac{12}{2} \frac{L}{120^2} \]

\[ E_{\text{buend}} = -F \theta = -FL \left( 1 - \frac{2}{6} \theta \sin \frac{\theta}{2} \right) \]

\[ E_{\text{tot}} = \frac{12}{2} \frac{\theta^2}{L} = FL \frac{\theta^2}{24} \]

\[ \theta^2 \left[ \frac{12}{L} - \frac{FL}{12} \right] \]

\[ \Rightarrow \text{instability at} \]

\[ F_c = \frac{12}{12L^2} \]

\[ \text{Note: the bent beam is not actually an arc but rather a sine} \]

\[ \Rightarrow F_c = \frac{12}{u^2 L^2} \]

Microhulse

\[ F_c = \frac{\rho \pi}{2L^2} \]