31/10/2017

Cytoskeleton

- actin : 7nm : two twisted protofilaments
- intermediate filaments : 10nm, coiled - coil
- microtubule : 25nm, hollow tube

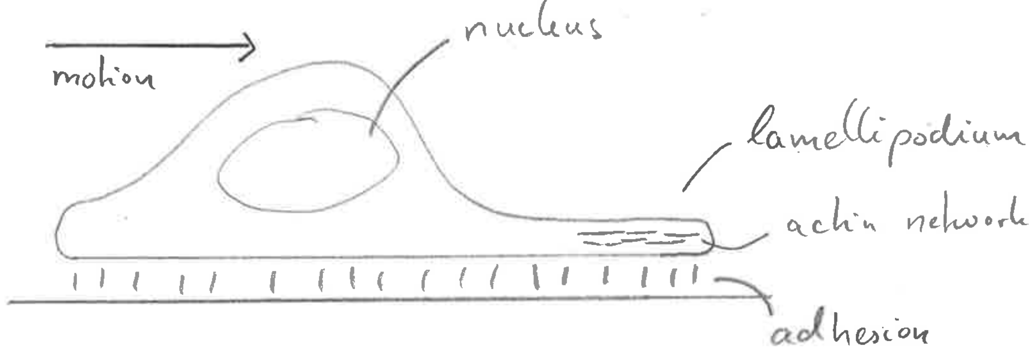
- Microtubuli

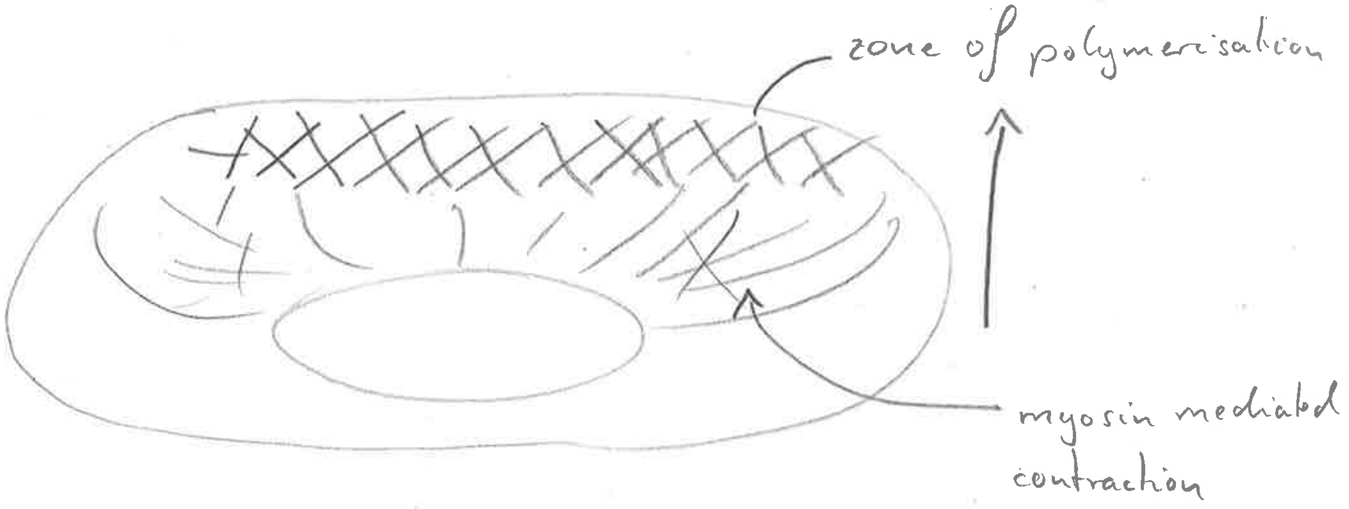
→ projector

- Actin

→ projector

- keratocyte mobility

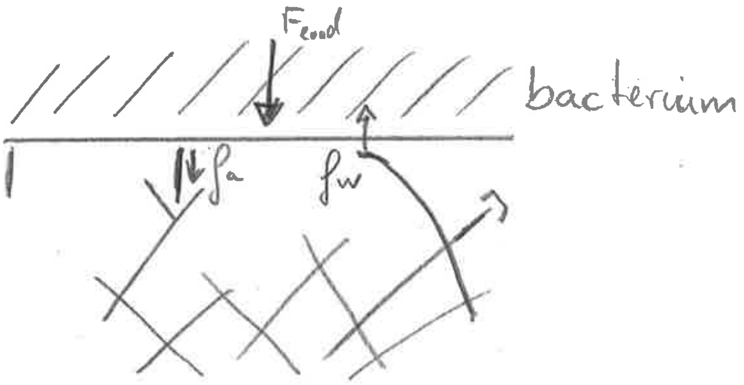




→ disassembly

- little myosin at the front
  - → force by polymerization
- myosin at the rear
  - → driving flux

Propulsion of listeria



$$F_{load} + a f_a = w f_w$$ ← **Force balance**

# of working filaments

# of attached filaments

nucleation

$$\frac{da}{dt} = n - \delta a \longrightarrow \bar{a} = \frac{n}{\delta}$$

dissociation

$$\frac{dw}{dt} = \delta a - \kappa w \longrightarrow \bar{w} = \frac{n}{\kappa}$$

capping

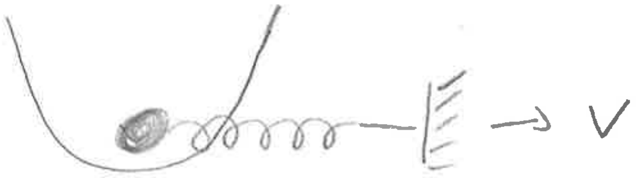
- how do rates depend on forces?

$$V = V_{max}\, e^{-f_w l / kT} - V_{dep}$$

reduction of polymerization due to force f_w

→ gap d is Boltzmann distributed

- dissociation is faster for large pulling forces
- as the cell moves, the force increases



soft spring with stiffness κ

$$f_a = kVt \Rightarrow \delta(V,t) = \delta_0 e^{kVt/f_b}$$

↑ f_b is a rupture scale of the attachment bond

- $$p(t) = \underset{\text{instantaneous rupture rate}}{\delta(t)} \underbrace{e^{-\int_0^t dt' \delta(t')}}_{\text{not ruptured yet}}$$

$$= \delta_0 e^{kVt/f_b} e^{-\int_0^t dt' \delta_0 e^{kVt'/f_b}} = \delta_0 e^{kVt/f_b} e^{\frac{\delta_0 f_b}{kV}[1-e^{kVt/f_b}]}$$

$$= -\frac{d}{dt} \underbrace{e^{\frac{\delta_0 f_b}{kV}[1-e^{kVt/f_b}]}}_{\text{not ruptured yet}}$$

- for $V > \frac{\delta_0 f_b}{\kappa} = V_0$
most ruptures happen when stretched
- $V < V_0 \Rightarrow$ unstretched
- $$p(t) = -\frac{d}{dt} e^{V_0/V [1 - e^{V/V_0 \delta_0 t}]}$$

- the average attachment force given $v = \frac{V}{V_0}$

$$f_a = kV\langle t\rangle = k v V_0 \int_0^\infty dt\, t \left[-\frac{d}{dt} e^{\frac{1}{v}[1-e^{v\delta_0 t}]}\right]$$

$$= f_b v \int dt\delta_0\, e^{\frac{1}{v}(1-e^{v\delta_0 t})} = f_b v\, \omega(v)$$

$$\approx \begin{cases} f_b v & v \ll 1 \\ f_b \log v & v \gg 1 \end{cases}$$

Putting things back together

$$V = V_{max}\, e^{-f_w l/kT} - V_{dep}$$

$$F_L + \frac{n}{\delta} f_a = \frac{n}{K} f_w \Rightarrow f_w = \frac{K}{n} F_L + \frac{K}{\delta} f_a$$

$$= \frac{K}{n} F_L + K \frac{f_b v\, \omega(v)}{\delta_0/\omega(v)}$$

$$\Rightarrow V = V_{max}\, e^{-\frac{Kl}{kT}\left[\frac{F_L}{n} + \frac{f_b \omega^2(v) v}{\delta_0}\right]} - V_{dep}$$

$$\varepsilon_1 = \frac{f_b l}{kT} \underbrace{\frac{x}{\delta_0}}$$

work in unit of kT

# of attached filaments per working filaments

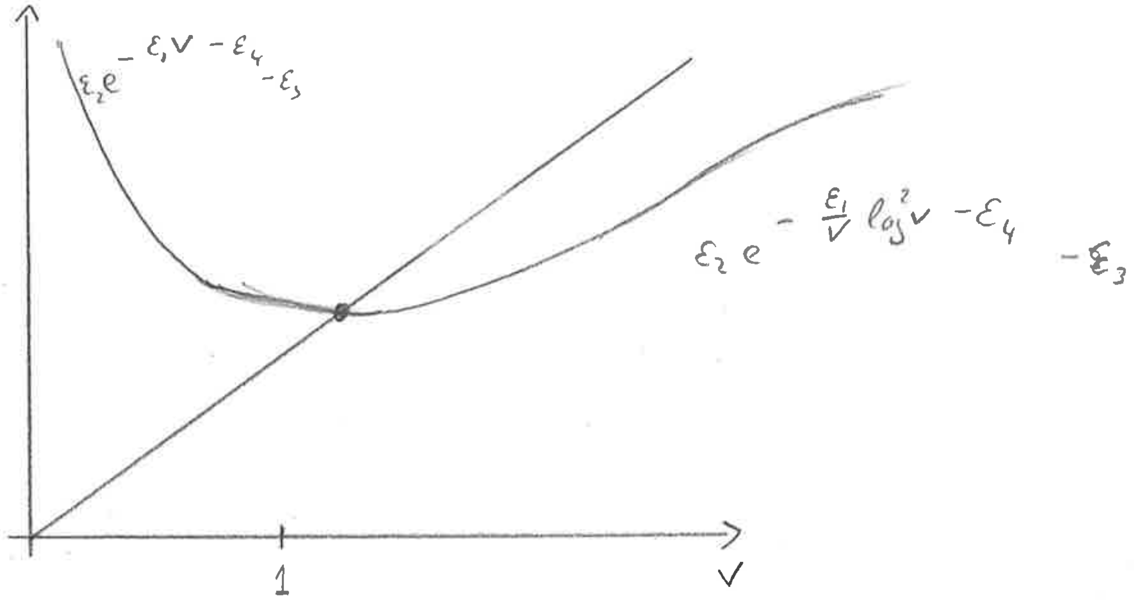
$$\varepsilon_2 = \frac{V_{max}}{V_0}$$

$$\varepsilon_3 = \frac{V_{dep}}{V_0}$$

$$\varepsilon_4 = \frac{F_L l}{kT} \frac{K}{n}$$

1/# of working filaments

$$\Rightarrow \boxed{v = \varepsilon_2\, e^{-\varepsilon_1 v\, \omega^2(v) - \varepsilon_4} - \varepsilon_3}$$



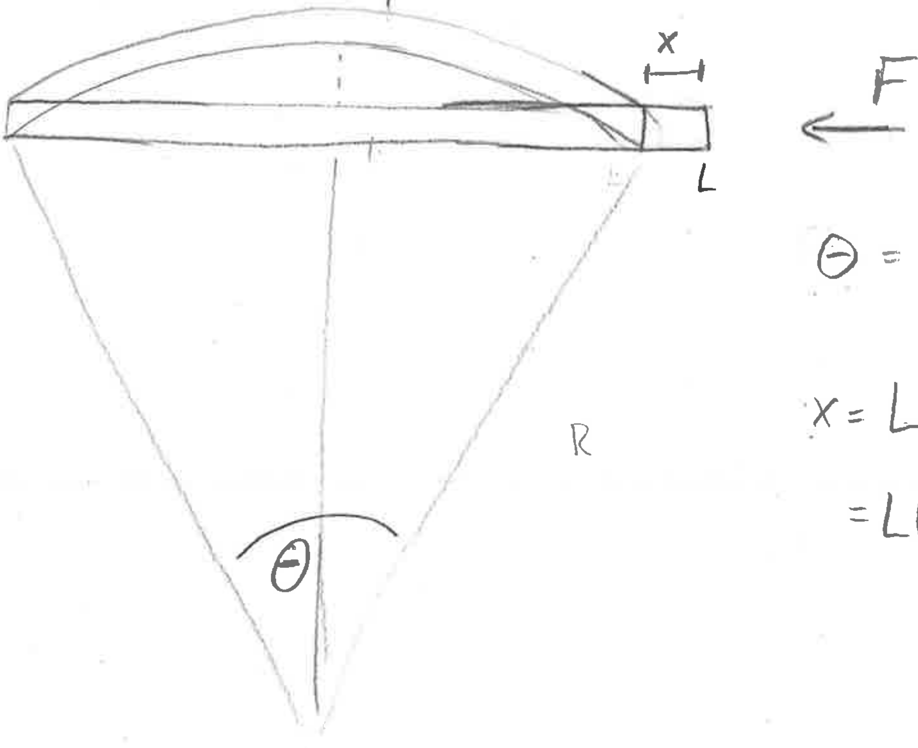
$\varepsilon_2 e^{-\varepsilon_4}$ scale the curve

ε_3 shifts the curve

Euler buckling instability

persistence length of microtubuli?

- $l_p = \frac{K}{kT} = 1\,\mathrm{mm} \Rightarrow$ stiffness $\kappa \simeq 4\,\mathrm{pN\,nm\,mm} = 4\,\mathrm{pN}\,\mu\mathrm{m}^2$
- how much compression can a microtubule support?



$$\Theta = \frac{L}{R}$$

$$x = L - 2R\sin\frac{\Theta}{2}$$
$$= L\left(1 - \frac{2}{\Theta}\sin\frac{\Theta}{2}\right)$$

$$E_{Bend} = \frac{\kappa}{2} L/R^2$$

$$= \frac{\kappa}{2} \Theta^2/L$$

$$E_{load} = -Fx = -FL\left(1 - \frac{2}{\Theta}\sin\frac{\Theta}{2}\right)$$

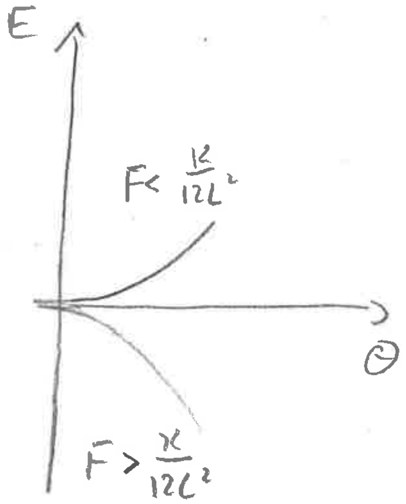
$$= -FL\left(1 - 1 + \frac{2}{\Theta}\frac{\Theta^3}{68}\right)$$

$$= -FL\frac{\Theta^2}{24}$$

$$E_{tot} = \frac{\kappa}{2}\frac{\Theta^2}{L} - FL\frac{\Theta^2}{24}$$

$$= \frac{\Theta^2}{2}\left[\frac{\kappa}{L} - \frac{FL}{12}\right]$$

→



⇒ instability at

$$F_c = \frac{\kappa}{12L^2}$$

__Note:__ the bent beam is not actually an arc but rather a sin

$$\Rightarrow \quad F_c = \frac{\kappa}{\pi^2 L^2}$$

Microtubule

$$F_c = \frac{pN\,\mu m^2}{3L^2}$$